Instructions: Complete each of the following exercises for practice.

1. Compute the domain of the vector function $\mathbf{r}(t) = \left\langle \ln(t+1), \frac{t}{\sqrt{9-t^2}}, 2^t \right\rangle$.

2. Compute the following limits.

   (a) $\lim_{t\to 0} \left(e^{-3t}\mathbf{i} + \frac{t^2}{\sin^2(t)}\mathbf{j} + \cos(2t)\mathbf{k} \right)$.

   (b) $\lim_{t\to\infty} \left\langle \frac{1+t^2}{1-t^2}, \arctan(t), \frac{1-e^{-2t}}{t} \right\rangle$

3. Sketch the curve with the given vector function. Indicate the direction in which t increases by an arrow.

   (a) $\mathbf{r}(t) = \langle \sin(t), t \rangle$

   (b) $\mathbf{r}(t) = \langle \sin(\pi t), t, \cos(\pi t) \rangle$

   (c) $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$

   (d) $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$

   (e) $\mathbf{r}(t) = \langle 3, t, 2-t^2 \rangle$

   (f) $\mathbf{r}(t) = t^2\mathbf{i} + t^4\mathbf{j} + t^6\mathbf{k}$

4. Find vector equation and parametric equations for the line segment joining $(2,-3,1)$ to $(6,2,-2)$.

5. Do Problems 13.1.21 through 13.1.26 in your textbook (matching some parametric equations to pictures).

6. Show that the curve $x = t\cos(t)$, $y = t\sin(t)$, $z = t$ lies on the cone $z^2 = x^2 + y^2$. Make a sketch of the curve.

7. At what points does the helix $\mathbf{r}(t) = \langle \sin(t), \cos(t), t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

8. Find a vector function tracing the curve of intersection of the surfaces given in $\mathbb{R}^3$.

   (a) $S_1 : x^2 + y^2 = 4, \quad S_2 : z = xy$

   (b) $S_1 : z = \sqrt{x^2+y^2}, \quad S_2 : z = 1 + y$

   (c) $S_1 : z = 4x^2 + y^2, \quad S_2 : y = x^2$

   (d) $S_1 : z = x^2 - y^2, \quad S_2 : x^2 + y^2 = 1$

   (e) $S_1 : x^2 + y^2 + 4z^2 = 4$ on $y \geq 0$, $S_2 : x^2 + z^2 = 1$

9. Two particles travel along the curves $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ below. Do they collide? Do their paths intersect?

   $$\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle \qquad \mathbf{r}_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle.$$

10. Suppose $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are vector functions and $f(t)$ is a scalar function. Prove the following.

    (a) $\lim_{t\to a} [\mathbf{u}(t) + \mathbf{v}(t)] = \left[\lim_{t\to a} \mathbf{u}(t)\right] + \left[\lim_{t\to a} \mathbf{v}(t)\right]$

    (b) $\lim_{t\to a} [f(t)\mathbf{v}(t)] = \left[\lim_{t\to a} f(t)\right] \left[\lim_{t\to a} \mathbf{v}(t)\right]$

    (c) $\lim_{t\to a} [\mathbf{u}(t) \cdot \mathbf{v}(t)] = \left[\lim_{t\to a} \mathbf{u}(t)\right] \cdot \left[\lim_{t\to a} \mathbf{v}(t)\right]$

    (d) $\lim_{t\to a} [\mathbf{u}(t) \times \mathbf{v}(t)] = \left[\lim_{t\to a} \mathbf{u}(t)\right] \times \left[\lim_{t\to a} \mathbf{v}(t)\right]$